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MICROWAVE AND ELECTRON BEAM COMPUTER PROGRAMS

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1720 Randolph Road SE
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June 1988

Final Report

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The computer programs CCUBE, ISIS, and GRADR were developed at Los Alamos National Laboratory (LANL). MAGIC, CPROP, IPROP, ORBIT, and KMRAD were written by MPC for Sandia National Laboratories (SNL), Albuquerque. SOS and BALTIC were written by MRC for the Air Force Weapons Laboratory (AFWL). IVORY was written by MRC for LANL. ARCTIC was written by MRC for the Strategic Defense Initiative Office (contract monitored by the Naval Surface Weapons Center, White Oak). BTRSQ was written by MRC for the Office of Naval Research (ONR). SCRIBE was adapted by MRC from the Stanford Linear Accelerator Center Beam Trajectory Program, EGUN.



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I. INTRODUCTION

Over the past several years, Mission Research Corporation (MRC) has developed two sets of general-purpose computer programs for multidimensional, electromagnetic simulations of relativistic beams and plasmas. Applications have included magnetic insulation, electron diodes, electron and ion acceleration, electron-layer magnetic compression, ion-focused transport of electron beams, beam-plasma instabilities, electron beam atmospheric propagation, and microwave generation. In addition, MRC has several simpler, faster running codes for investigating electron beam equilibrium and linear stability properties. For the most part, these smaller codes have been employed in accelerator research. The simulation codes, together with the more important equilibrium and stability codes, are listed in Table 1.

This report summarizes the capabilities and describes the physical models of these codes, especially as they bear upon microwave source investigations. Section II presents briefly the principal capabilities of each of the fourteen codes in Table 1. Section III begins with an overview of particle-in-cell (PIC) simulation techniques and then enumerates the unique aspects of each of the general purpose PIC simulation codes. The models employed in the remaining computer programs are discussed in Section IV.

All programs listed in Table 1 except CPROP run on CRAY-1 computers under the CTSS operating system. As noted in Section II, some run on other computers as well. Copies reside at the Los Alamos National Laboratory (LANL) computer center and elsewhere. The U. S. Government either owns or has a royalty-free license to use any of the programs.

TABLE 1. MAJOR MRC COMPUTER PROGRAMS FOR MICROWAVE
AND CHARGED PARTICLE BEAM RESEARCH

<u>CODE</u>	<u>APPLICATIONS</u>	<u>REFERENCE</u>
CCUBE*	2-D PIC Microwave and Beam Simulations	1
IVORY	3-D PIC Microwave and Beam Simulations	2
ISIS*	2-D PIC Microwave and Beam Simulations	3
MAGIC	2-D PIC Microwave and Beam Simulations	4
SOS	3-D PIC Microwave and Beam Simulations	5
CPROP	2-D PIC Beam Atmospheric Propagation	6
IPROP	3-D PIC Beam Atmospheric and IFR Propagation	7
SCRIBE	2-D Electron Diode Equilibria	8
ARCTIC	2-D Ion-Focused Transport Equilibria	9
BALTIC	Beam Transverse Stability in Accelerators	10
GRADR*	Laminar Beam Stability, Microwave Growth	11
ORBIT	1-D Beam Kinetic Equilibria	12
KMRAD	Kinetic Beam Stability, Microwave Growth	13
BTRSQ	Recirculating Accelerator Instabilities	14

*Developed by Los Alamos National Laboratory and used with its permission.

II. CODE CAPABILITIES

1. OVERVIEW

PIC codes treat the detailed electromagnetic interactions among one or more plasmas, charged particle beams, and metallic structures. They have been employed fruitfully in every branch of plasma physics from fusion energy to astrophysics; the linear and nonlinear evolution of instabilities is a common theme. In high-current electron accelerator research, PIC codes are used to treat pulsed-power flow in magnetically insulated transmission lines, electron beam generation in complex diodes, beam transport and inductive acceleration in vacuum drift tubes, and beam transport in low density ion channels. Beam propagation in partially ionized gases is a related application. High-power microwave sources are modeled effectively by PIC codes, which can follow simultaneously the electron beam nonlinear dynamics and the resonant electromagnetic response of the microwave cavity. Microwave extraction also can be examined. The microscopic treatment provided by PIC codes is especially valuable in verifying the frequency and phase coherence of extended microwave sources. PIC codes also can be utilized in microwave antenna design, propagation analysis, and coupling studies, although simpler codes typically are preferable.

2. CCUBE

The PIC code CCUBE is used extensively to model microwave generation (Refs. 15-18) and beam acceleration (Refs. 19-22). Although only two-dimensional, the code is very flexible in that it can run in any orthogonal coordinate system. Injection and extraction of particles and fields at boundaries allows it to treat complicated, open-ended microwave devices. The code has extensive diagnostics, including particle and field plots, time histories of various quantities, and color movies. A graphics post-processor package, PEGASUS, is available (Ref. 23). CCUBE runs on VAX 780 and CONVEX computers, as well as on CRAY-1 computers.

3. IVORY

IVORY is a three-dimensional generalization of CCUBE. Field variation in the third dimension is treated by Fourier series of one to about six terms. As a consequence, the code is best suited for problems in which the particle dynamics are only weakly nonlinear in the third dimension and boundary conditions are approximately two-dimensional. Examples include free electron lasers, many microwave sources, particle beam transport (Refs. 24-27), and plasmoid propagation across a magnetic field (Ref. 28). IVORY is very efficient for such problems, requiring computer resources only a few times greater than those needed for corresponding two-dimensional problems.

4. ISIS

ISIS is a significantly enhanced, but still two-dimensional, version of CCUBE. Special features include Monte Carlo transport of high energy particles, volume creation of particles by high energy electrons and photons, and a hydrodynamic treatment of imploding resistive liners. The layout of complicated electrode geometries is largely automated, and a new algorithm for improved treatment of curved metallic surfaces is under development. It and CCUBE are used at LANL in vircator (Refs. 29-30), electron diode, collective acceleration, ring compression, and beam-plasma interaction studies.

5. MAGIC

MAGIC is a two-dimensional PIC code quite similar in capabilities to CCUBE but developed independently. It is employed at Sandia National Laboratories (SNL) and elsewhere for magnetic insulation, electron diode (Ref. 31), collective acceleration (Ref. 32), beam transport (Ref. 33), and microwave generation studies (Ref. 34).

6. SOS

SOS is a fully three-dimensional, general purpose simulation code. It accommodates essentially arbitrary boundary conditions in cartesian or cylindrical coordinates. Wave and particle emission and absorption, metallic structures, dielectrics, and air conductivity packages are included. Sophisticated data management techniques allow it to treat problems of immense size and complexity, limited only by the availability of computer time. A compact, "user friendly" data entry system simplifies problem specification. In addition to plasma physics research, SOS often is used in nuclear weapons effects studies. SOS runs on VAX 780 and CONVEX computers, as well as on CRAY-1 computers.

7. CPROP

CPROP is an enhanced and optimized version of CCUBE for axisymmetric investigations of electron beam propagation in air. Special features include a mesh which moves with the beam, a partially implicit field solver to permit large time steps, a multispecies air chemistry package, and a delta-ray creation and destruction procedure. CPROP is not limited by the paraxial particle and frozen field approximations used in many other propagation codes. The code has been employed in Nordsieck expansion (Ref. 35), beam front erosion, and hollowing instability studies supporting RADLAC (Refs. 36-37), IBEX (Refs. 38-40), PHERMEX (Ref. 41), and FX-100 (Ref. 42) experiments. CPROP runs on CDC-7600 computers under the LTSS operating system.

8. IPROP

IPROP is a three-dimensional electron beam atmospheric propagation code obtained by adding the special features of CPROP to IVORY. Diamagnetic effects are included to treat the RADLAC beam rotation properly. The code can model all resistive instabilities -- hollowing, hose, and filamentation -- either separately or in combination (Ref. 36). Two major channel-tracking discoveries were made with IPROP in 1986 (Refs. 43, 44). With the air

conductivity package omitted, IPROP also is used for research on electron beam transport in low-density ionized channels, the so-called ion-focused regime (IFR) (Ref. 45). Research centers on beam front erosion, two-stream instabilities, and channel electron trapping.

9. SCRIBE

SCRIBE can be described as a two-dimensional, steady-state PIC code. It is based on Hermannsfeldt's Electron Trajectory Program (Ref. 8). It computes axisymmetric electron flow patterns in self-consistently determined static electric and magnetic fields. Being time-independent, the code is fast and relatively free of numerical noise. SCRIBE is ideal for studying electron equilibrium behavior in diodes (Ref. 46), accelerators and microwave tubes. A particularly noteworthy achievement with SCRIBE is the design of very low emittance electron diodes and transport systems for free electron laser studies at the Naval Research Laboratory (Ref. 47). SCRIBE runs on VAX 780 computers, as well as on CRAY-1 computers.

10. ARCTIC

ARCTIC is similar to SCRIBE but runs in the beam frame and solves the frozen field equations. It was written to determine electron and ion trajectories and associated steady-state fields for relativistic electron beam propagation in IFR channels. For such applications ARCTIC is far more efficient than alternative codes, especially when very long beam pulses and large drift tubes are involved.

11. BALTIC

BALTIC is a simple code used to follow the transverse oscillations of the centroid of a particle beam in an accelerator. Specifically, it computes the linear growth of the resistive wall (Ref. 48), image displacement (Ref. 24), and beam breakup instabilities (Refs. 10, 49) for long pulse beams in multigap accelerators like RADLAC. With minor modifications it could be employed to predict gain in Scantron devices.

12. GRADR

GRADR is a general-purpose equilibrium and linear dispersion code for laminar, cylindrically symmetric, radially inhomogeneous particle beams. Conducting boundaries and resistive and dielectric mediums are allowed. Starting from user-supplied constraint equations, the code first computes the beam equilibrium current and field profiles. It then determines wave frequencies and growth rates for arbitrary, user-requested wave numbers. GRADR is interactive and exceedingly fast. Typically, the code is used in collective acceleration (Refs. 19, 50-52) and beam stability studies (Refs. 48, 53-56). With modifications, it could be employed for microwave generation in slow wave structures (Ref. 57).

13. ORBIT

ORBIT and KMRAD are used in place of GRADR for nonlaminar beams, the more common situation. ORBIT determines the equilibrium radial profiles of the particle currents and fields, based on a user-provided subroutine specifying the particle distribution function in terms of constants of the motion. A family of distribution function subroutines for relativistic Bennett beams and for radial diodes is available (Ref. 58).

14. KMRAD

KMRAD is a three-dimensional linearized PIC code. A cylindrically symmetric, radially inhomogeneous particle distribution is specified by the user, possibly with the aid of ORBIT. (An interface between these codes is provided.) It then computes the temporal evolution of perturbed particle orbits and electromagnetic fields for arbitrary wave numbers chosen by the user. The code is interactive and moderately fast. KMRAD is used regularly for parametric studies of instability growth for propagating particle beams in air (Refs. 36, 59-61) and in IFR channels. Smooth-bore magnetron, Cherenkov masers, gyrotrons, and orbitrons also can be studied. With modifications, the code can be employed for microwave generation in slow wave structures.

15. BTRSQ

BTRSQ is the most detailed of a family of dispersion-relation solvers for high-current betatrons (Refs. 25, 62). The beam is treated in the rigid disk limit, while the fields are determined exactly for a cavity of rectangular minor cross section. The code is interactive and exceedingly fast. Many new properties of the negative mass instability in modified betatrons have been discovered recently with BTRSQ (Refs. 14, 27, 63).

Dispersion relation codes for microwave generation can be developed rapidly as the needs arise. A convenient shell, developed originally for GRADR, provides user interaction, error recovery, a Muller's method (Ref. 64) complex root evaluator, and a graphics interface. The programmer merely writes a dispersion function evaluation routine for the new device. For instance, the Pierce diode electromagnetic stability code PEM2D being written for AFWL by MRC uses this shell.

III. PIC CODE ALGORITHMS

1. GENERAL FEATURES OF PIC SIMULATION CODES

PIC codes determine the time evolution of complex plasmas by computing the dynamics of many thousands of representative plasma particles (electrons and/or ions) moving in electromagnetic fields externally applied or produced by the plasma itself. Thus, PIC codes provide the most fundamental and detailed representation possible of plasma problems. In effect, they solve the Vlasov equation. Of course, this precision comes at the cost of substantial computer requirements, and for this reason PIC codes should be employed only when simpler numerical or analytical techniques are inadequate.

The electromagnetic fields are defined on a regular mesh in one, two, or three dimensions, depending on the symmetry of the problem to be solved. The mesh can be in rectangular, cylindrical, or other desired geometry. At each time step, new electric (E) and magnetic (B) fields are computed by advancing the finite difference approximations to Maxwell's equations,

$$\frac{\partial \mathbf{E}}{\partial t} = \nabla \times \mathbf{B} - \mathbf{J} \quad (1)$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \quad (2)$$

using currents (J) determined from the plasma particle motion on the previous time step. Alternatively, equations for the scalar and vector potentials can be solved. Boundary conditions are required to define spatial derivatives at the mesh edges. Wave reflecting (i.e., metallic) or periodic boundary conditions are common choices. More complicated boundaries allow electromagnetic waves to be launched into the computational region or to leave it. Metal structures at locations within the computational region are represented simply by setting electric field components to zero there. Representing dielectrics, resistors, and simple driven antennas is only slightly more difficult.

Particle momenta ($P = \gamma V$), positions (X), and energy (γ) are then advanced using the relativistic equations of motion with the newly computed fields.

$$\frac{dP}{dt} = E + V \times B \quad (3)$$

$$\frac{dX}{dt} = V \quad (4)$$

$$\gamma = (p^2 + 1)^{1/2} \quad (5)$$

(Replace γ by 1 for nonrelativistic problems.) The fields appearing in Equation 3 are those at the particle location, obtained from the fields at nearby mesh points by (typically, linear) interpolation. When a particle leaves the computational region, it is destroyed or it is returned to the mesh by some prescribed procedure (e.g., reflected). By the same token, particles can be injected from boundaries, a feature particularly useful in particle beam simulations. After a particle's new position and momenta have been determined, its contribution to the plasma currents is obtained by interpolating $V = P/\gamma$ to nearby mesh points.

This cycle of advancing fields based on particle currents and then advancing particles based on the new fields is repeated hundreds of times in a typical simulation. The time step is set by the smallest time scale in the calculation, which may be the plasma oscillation period, the electron cyclotron period, or the Courant time (of order the time for a light wave to cross a cell in the mesh). Progress has been made in the last few years at surmounting the Courant limit, which is numerical rather than physical in character (Ref. 65). Cell dimensions must be small compared to spatial scales of interest.

The PIC code running times and memory requirements are highly problem-dependent. The CPU times on a CRAY-1 computer typically range between 15 min and 4 h, although 20-h runs are not unheard of. Corresponding central memory needs vary between $2 \cdot 10^5$ and $4 \cdot 10^6$ words. At least two fast, large capacity disks also are needed. Historically, the physics problems attempted with PIC

codes have expanded to consume the maximum resources available in each generation of computers.

PIC codes usually have extensive graphics output capabilities and operating-system interfaces. Advances in PIC technique are described in several books (Refs. 66-67).

2. CCUBE

CCUBE (Ref. 1) is a general-purpose two-dimensional PIC code, and as such operates along the lines described above. It does, however, have a number of special features which increase its flexibility. The Galerkin finite-element algorithm for advancing particle quantities (Ref. 68) eliminates the numerical Cherenkov instability (Ref. 69) and ameliorates some other numerical problems. The electromagnetic fields incorporate backward biasing to damp unwanted high-frequency field fluctuations and to further suppress possible numerical instabilities (Ref. 70). The MRC-developed time-biased field solver also can significantly relax the Courant time step constraint, reducing computing costs. Marder's procedure for reducing charge conservation errors is being added (Ref. 71). CCUBE is written for arbitrary orthogonal coordinates. The user can convert from, say, cylindrical to toroidal coordinates merely by revising routines which define metric elements. Nonuniform zoning of the spatial mesh is accomplished with equal ease.

A number of other features deserve mention. For compatibility with nonuniform zoning, variable particle weighting within particle type, with several particle species permitted, is implemented. The code supports periodic, wave transmitting, and inhomogeneous Dirichlet-Neumann field boundary conditions. Particles can be absorbed, reflected, or injected from surfaces not necessarily coinciding with edges of the mesh. The physics for wave launching and particle field-emission from surfaces, which is necessary to accurately model microwave and particle beam sources, is incorporated. Graphics output can be produced either directly from CCUBE or interactively from the PEGASUS (Ref. 23) postprocessor. Options include microfiche plots of

various slices through particle phase space, particle distribution functions, contour plots and one-dimensional cuts of fields and currents, and histories of particle and field energies and other selected quantities. Color movies of particle and field data can be generated concurrently. In addition to the more traditional simulation diagnostics just mentioned, CCUBE contains numerical equivalents of such experimental diagnostics as Faraday cups, calorimeters, compensated diamagnetic loops, Rogowski coils, and local probes for field and current measurements. Each is time dependent and can be Fourier analyzed.

3. IVORY

IVORY (Ref. 2) is a three-dimensional generalization of CCUBE and shares with it most of the special features described above. For economy of operation and clarity of interpretation, variations of field quantities in the third (usually, azimuthal) coordinate are represented by spectral rather than finite difference methods; i.e., the fields are Fourier decomposed in that dimension. In this way the behavior of selected modes can be examined without wasting time and storage on other, uninteresting modes. It should be emphasized that the spectral method is not a linearization in either particle or field behavior. Of course, the greater the nonlinearity of a problem, the more azimuthal modes are needed to resolve it accurately.

Thus, IVORY is particularly suitable for configurations which depart only weakly from symmetry in one coordinate, such as a particle beam in an accelerator or microwave device. The code provides three-dimensional results at costs comparable to two-dimensional simulations in such cases.

4. ISIS

The two-dimensional ISIS code (Ref. 3) also is a derivative of CCUBE, modified to allow more accurate treatment of particle beam interaction with materials. Electron- and photon-generated delta rays can be created, transported, and destroyed in the simulations. A Monte Carlo treatment of Moliere

scattering is provided for particles passing through foils and other materials. Energy loss is modeled by Bethe's continuous-slowing-down approximation. A resistive magnetohydrodynamic representation of metallic shells is useful in magnetic compression studies. Bulk resistivity terms have been added to the field equations.

Several numerical enhancements are noteworthy. Bilinear interpolation of currents and fields is permitted in addition to the Galerkin scheme. A current lay-down procedure explicitly guaranteeing charge conservation is provided. The backward-biased field algorithm is inverted exactly rather than iteratively. The algorithm for space-charge-limited particle emission from electrodes is more accurate, and an improved treatment of curved electrodes is being developed. The procedure for specifying the locations of complex electrodes on the mesh is automated.

5. MAGIC

The two-dimensional code MAGIC (Ref. 4) was developed almost entirely independently of CCUBE. Nonetheless, the two are very similar not only in their features but also in their internal algorithms.

Differences are noted here. Field interpolation in MAGIC is bilinear, and current interpolation is explicitly charge-conserving. Triangular zoning at conductor surfaces improves field accuracy there. Wave absorption at boundaries by graded electric and magnetic resistivity is accommodated. Input data processing is a bit more flexible. There is no postprocessor.

6. SOS

SOS (Ref. 5) is a general purpose three-dimensional PIC code. Unlike IVORY, it treats field variations in all three coordinates by finite differences. The code was originally developed for system-generated electromagnetic pulse computations but has been generalized to treat particle beam, microwave, and magnetic insulation problems.

A distinguishing feature of SOS is its data management procedure. At each time step, the code processes the three-dimensional mesh and the particles on it as a sequence of two-dimensional slices. Only a few contiguous slices are in computer memory at any instant, while the rest are stored on disk. (In most other PIC codes, fields and currents are retained entirely in computer memory, while particles are cycled between disk and memory.) In this way, extremely large computations can be performed, limited only by budget.

SOS has an X-ray-generated Compton electron creation routine. Conductivity generation by Compton electrons and other particles is determined using a three-species lumped air chemistry model with direct ionization, avalanche, and recombination. Bulk Compton currents and conductivity in dielectrics are treated by a special model. Exponential integrating factors in the field algorithm stably accommodate arbitrarily variable conductivity. A family of subgrid models is provided for structures too small to be represented accurately by standard finite differences. Other features of the codes are similar to those in MAGIC.

7. CPROP

CPROP (Ref. 6) is a two-dimensional PIC code designed for investigating the axisymmetric dynamics of high-current relativistic electron beam propagation in air. It is based on CCUBE and thus possesses most of the diagnostic and other capabilities of that program. Three major enhancements were required to treat beam transport in air: an electromagnetic field solver accommodating arbitrary scalar conductivity and a moving coordinate mesh, a conductivity generation package, and a Moliere electron scattering routine. CPROP also contains a high energy delta-ray generation and transport package.

The CPROP field algorithm is unique in several respects (Ref. 40). Calculations are carried out in a frame moving axially at a user-specified, possibly time-varying velocity. The field evaluation is partially implicit, so that the Courant limit is set only by the axial cell size. The so-called frozen field approximation (Refs. 43, 72) to Maxwell's equations can be

treated as a special case, if desired. Exponential integrating factors, similar to those in SOS, accommodate arbitrary scalar conductivities. Zoning is nonuniform both axially and radially.

Both the BMCOND (Ref. 73) and the PHOENIX (Ref. 74) air chemistry models are available. The former solves rate equations for gas electron density and temperature and for the densities of several ionic species. The latter is a lumped parameter model similar to that in SOS. Moliere scattering (Refs. 35, 75) is implemented in CPROP by applying a deflection to the beam electrons every few time steps. The deflection angle is chosen randomly from a set of previously computed small angles forming a truncated Moliere distribution or from an analytical expression for occasional large angles.

8. IPROP

IPROP (Ref. 7) is a version of IVORY into which most of the special features of CPROP have been transplanted. A new lumped parameter conductivity model calibrated against the SAIC HICHEM code is used (Ref. 76). Features of the electromagnetic field solver and the conductivity package were essential to recent MRC channel tracking discoveries (Refs. 43-44).

IV. ALGORITHMS OF OTHER MRC CODES

The remaining computer programs are much faster than PIC codes. SCRIBE, ARCTIC, BALTIC, and KMRAD typically use no more than a few minutes of CRAY-1 computer time; GRADR, ORBIT, and BTRSQ only seconds. None of these codes needs more than 10^5 words of memory.

1. SCRIBE

When only a steady-state simulation of electron beam generation and transport is needed, great economy can be achieved with SCRIBE. It is a version of the Stanford Linear Accelerator (SLAC) code EGUN (Ref. 8), extensively modified by MRC for research on intense beam diodes. SCRIBE self-consistently solves two-dimensional problems (usually in (r,z) geometry) by an iterative method. First, a solution to Laplace's equation is obtained, then current emission is calculated, and the electron trajectories are integrated using the fields. The trajectories deposit space charge at the mesh points, and a solution to Poisson's equation is calculated. Current emission is then recalculated and the cycle repeated until the potentials and trajectories are unchanged from cycle to cycle. Improved algorithms assure rapid convergence. All relevant self and applied electric and magnetic fields (except the beam-generated axial magnetic field) are included in the calculations. Both Dirichlet and Neumann boundaries are available, and surfaces are not restricted to lie along the grids.

2. ARCTIC

ARCTIC is a newly developed two-dimensional code for determining the steady-state flow of channel electrons and ions for IFR beam transport problems. At present, the beam is treated as a rigid rod propagating at approximately the speed of light. Calculations are performed in the beam frame. Representative channel electron and ion trajectories are computed based on self-consistent fields, and the corresponding currents interpolated onto a

spatial mesh. Because fields are evaluated in the frozen field limit (Refs. 43, 72), no iterations are required to reach the solution. In many respects, ARCTIC resembles a one-dimensional PIC code in which the axial coordinate takes the place of time.

A three-dimensional version of ARCTIC is planned for the future. Representing the beam cross section by an envelope model also is attractive, although iterative solution of the resulting system of equations would then be necessary.

3. BALTIC

BALTIC (Ref. 10) provides a less detailed but immensely faster treatment than IVORY of transverse beam instabilities in accelerators. Presently, it evaluates the simultaneous evolution of resistive wall (Ref. 48), image displacement (Ref. 24), and beam breakup (Refs. 10, 49) instabilities in multiple-gap linear induction and radio frequency accelerators. Arbitrary spatial variations in the beam energy and axial magnetic field strength are allowed. Provision exists for external forces. Adding centrifugal force and a vertical magnetic field for recirculating accelerator studies is straightforward.

BALTIC employs a number of simplifying assumptions for computational efficiency. The beam is represented as a chain of rigid disks propagating forward at a uniform speed. Electric and magnetic fields entering into the equations of transverse motion are computed in the long wavelength limit, and accelerating gaps are treated in the thin lens approximation. Coupling coefficients entering into various instabilities can be estimated analytically (Ref. 77) or determined from experiments or more detailed numerical calculations.

4. GRADR

GRADR (Ref. 11) determines the equilibrium profiles and linear eigenmodes of cylindrically symmetric, radially inhomogeneous, relativistic electron beams in the laminar flow approximation. The cylindrical equilibrium is evaluated from six nonlinear, coupled, algebraic and first-order differential equations. Four follow from Maxwell's equations plus the fluid equations, while two can be specified arbitrarily to select a desired equilibrium (Ref. 78). The two constraint equations should, of course, be physically realizable. One constraint almost always is appropriate: total particle energy, kinetic plus potential, must be constant across the beam and equal to the injection energy. As a second constraint, typically we let the current density profile of the beam be specified at injection. Conservation of canonical angular momentum then determines the current density profile within the accelerator drift tube. The resulting system of equations is solved iteratively.

Given an equilibrium determined in this or any other way, GRADR then solves the corresponding linearized (small amplitude wave) equations to obtain eigenmodes and eigenvalues. The linear equations form a fourth-order differential system in radius and are integrated by a Fehlberg fourth-fifth-order Runge-Kutta routine (Ref. 79). Muller's method is employed in finding wave frequencies and growth rates (Ref. 64). The options of computing wave energy and the adiabatic variation of wave amplitude with changing beam parameters are available (Ref. 51). Including background plasma conductivity in GRADR allows study of laminar beam resistive instabilities in air (Ref. 56). Modifying GRADR boundary conditions to accommodate slow-wave structures is straightforward (Ref. 57) and would allow gain calculations for many microwave devices.

5. ORBIT

ORBIT (Ref. 12) calculates one-dimensional, radially inhomogeneous, cylindrically symmetric beam and plasma equilibria. The code solves the relativistic Vlasov-Maxwell equations for user-supplied particle distribution

functions. Distributions are specified in terms of particle constants of motion: canonical axial momentum, canonical angular momentum, and energy. The kinetic framework includes finite temperatures in a natural manner and allows investigation of phenomena associated with the detailed momentum-space distributions that cannot be treated using a macroscopic fluid description.

ORBIT has been used primarily for investigating equilibria in relativistic electron beams and magnetically insulated diodes. Relativistic electron beam equilibria with both sharp and smooth edges have been examined, and distribution functions for hollow beams identified. For magnetically insulated diodes, both axial and azimuthal magnetic fields have been examined. Finite temperature effects, gas prefill, and electron injection with nonzero energy have been investigated. The use of ORBIT has eliminated making many of the assumptions and approximations necessary in the analytic treatment of magnetic insulation. A catalog of distribution functions and their properties has been compiled for use with ORBIT (Ref. 58).

6. KMRAD

KMRAD (Ref. 13) is a linearized but fully relativistic and electromagnetic linear stability code for arbitrary cylindrically symmetric plasma systems. It consists of two parts: a one-dimensional, radially resolved, nonlinear PIC code; and a three-dimensional, spectrally resolved, linear PIC code. The one-dimensional routines determine particle orbits and corresponding fields in a cylindrically symmetric, possibly slowly evolving, equilibrium. It can be initialized from ORBIT, and an interface exists between the codes.

The three-dimensional routines compute the temporal growth of small amplitude perturbations of the particle equilibrium orbits, and the fields arising from these perturbations. The three-dimensional fields are Fourier decomposed axially and azimuthally, and one mode at a time is analyzed. The instability growth rate of the fastest unstable wave of a selected mode-number pair (k_z, m) can be determined from the exponential growth of field energy in

the simulation. User-specified radially inhomogeneous background conductivity and return current profiles allow KMRAD to treat resistive instabilities. The code is interactive and quite fast. In effect, it simulates three-dimensional linear dynamics but requires computer resources comparable to those of a one-dimensional code. KMRAD plus ORBIT constitutes a kinetic beam generalization of GRADR. Like GRADR, it can be modified to handle microwave growth in slow wave structures.

7. BTRSQ

BTRSQ (Ref. 14) calculates the normal mode frequencies and growth rates, if any, of a particle beam in a modified betatron or similar device. It does so by numerically evaluating the roots of an analytical dispersion relation. The dispersion relation was derived by treating the beam as a string of rigid disks traveling around the toroidal acceleration cavity while executing small toroidal and poloidal oscillations. The electromagnetic fields which interact with these oscillations were evaluated exactly for a cavity of rectangular minor cross section by means of Green's functions.

Many features of BTRSQ are patterned after those in GRADR. It has interactive NAMELIST input, an optimized Muller's method root finder, error recovery and user-interrupt procedures, and graphical output. It is extremely fast and very easy to use. Other linear stability problems can be addressed in the same way simply by providing the appropriate dispersion function routines. This has been done in several instances.

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